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Geological Survey
Suirbhéireacht Gheolaíochta
Ireland | Éireann

Research Programme
- Short Call 2020, Final Report -

Lead Applicant Name	Ivan Lokmer
Host Organisation name	University College Dublin
Project Title	Amplification of seismic rotations due to subsurface heterogeneities
Contract Number	2020-sc-034

This report covers the period 1st December 2020 to project end date 6th December 2021

2. (a) Project information:

Final project report (*max 2500 words, excluding figures and headings*):

Executive summary (lay abstract, approx. 250 words):

The seismic waves responsible for shaking civil engineering structures undergo interference, focusing, scattering, and diffraction by the inhomogeneous medium encountered along the source-to-site propagation path. The subsurface heterogeneities at a site can particularly alter the local seismic wavefield and amplify the ground rotations, thereby increasing the seismic hazard. This project aimed at quantifying these amplifications in ground rotations in the case of a 2-D heterogeneous, semi-infinite elastic medium excited by plane SH waves. The heterogeneities in the medium are described through fluctuations in the shear modulus and density superimposed on a homogeneous background elastic medium. A semi-analytical method based on the perturbation theory is developed to obtain the Fourier spectra of the resulting translational and rotational ground motions. In this method, the problem of simulating motion in a heterogeneous medium is reduced to calculating the response of a homogeneous medium subjected to body forces representing the heterogeneities. Since the dynamic response of a homogenous elastic half-space subject to body forces is easy to synthesize, the proposed method is convenient to implement and is computationally less intensive than numerical techniques such as finite-difference, finite-element, etc. The method is tested for accuracy by comparing its solution with that of a spectral finite element-based solver. Furthermore, the method is shown to be stable at high frequencies (>1 Hz) as well as when the subsurface heterogeneities are strong (~20%). The method is applied to an example 2-D heterogeneous medium to ascertain the amplifications in the ground rotations.

(i) Results, scientific/engineering targets reached beyond the state of the art and conclusions

Problem description

We are evaluating the impact of subsurface heterogeneities on rotations resulting from the incidence of SH waves. For this, we have considered the problem of plane SH waves travelling in a 2-D homogeneous, elastic half-space with embedded heterogeneous elastic region (see Figure 1). The heterogeneity in this region is characterized by spatially varying shear modulus and density,

$$\mu(x_1, x_3) = \mu_m + \epsilon\mu_d(x_1, x_3)$$
$$\rho(x_1, x_3) = \rho_m + \epsilon\rho_d(x_1, x_3) \tag{1}$$

where μ_m and ρ_m respectively denote the shear modulus and density of the background homogeneous region and $\epsilon\mu_d(x_1, x_3)$ and $\epsilon\rho_d(x_1, x_3)$ denote the deviation in these properties, controlled by the perturbation parameter ϵ.

The SH waves in this medium give rise to out-of-plane translational motion u_2 and torsional motion Ω_3 (i.e., rotation about vertical axis) at the stress-free surface $(x_3 = 0)$.

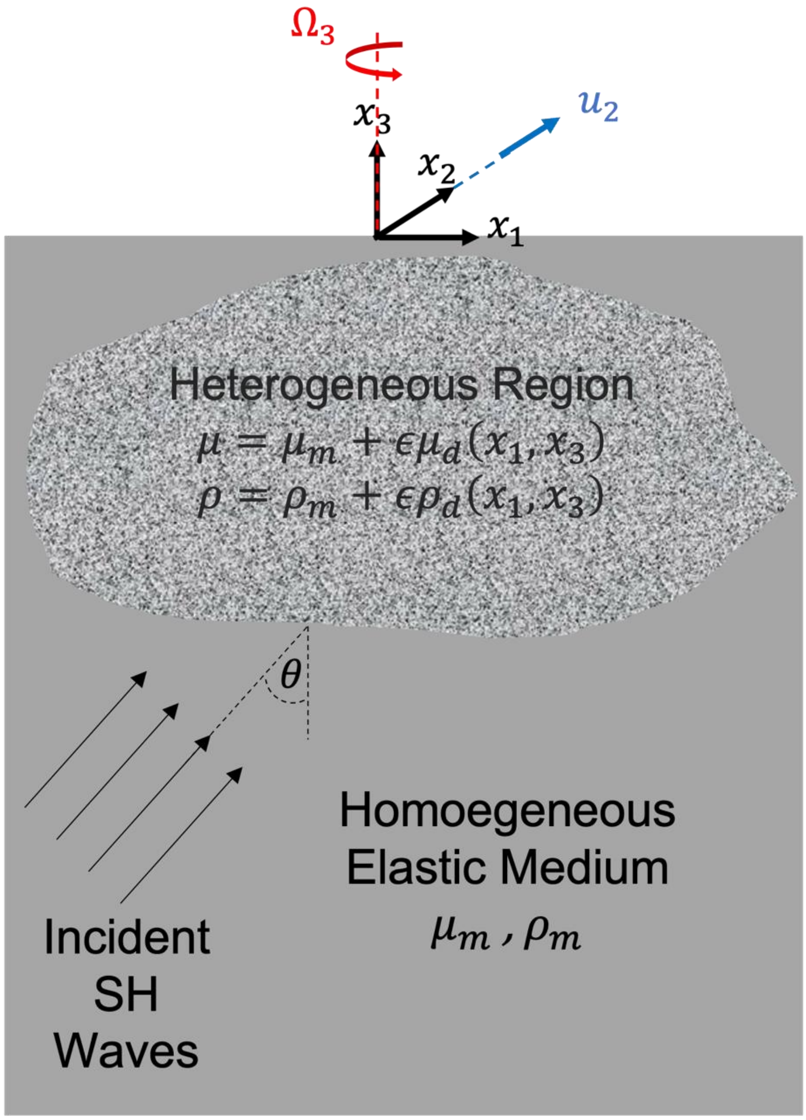

Figure 1: Seismic wave propagation problem considered in this project.

Methodology

To simulate the ground motions (rotations and translations) for the above-described heterogeneous medium, a semi-analytical approach based on perturbation theory has been developed. In this method, the 2-D wave propagation problem is considered in the frequency (ω) domain

$$\frac{\partial}{\partial x_1}\left(\mu\frac{\partial u_2}{\partial x_1}\right)+\frac{\partial}{\partial x_3}\left(\mu\frac{\partial u_2}{\partial x_3}\right)+\rho\omega^2u_2=0 \tag{2}$$

together with the following solution of the translational wavefield:

$$u_2(x_1,x_3,\omega)=u_m(x_1,x_3,\omega)\ +\sum_{r=1}^{\infty}\epsilon^r\,u_d^{(\mathrm{r})}(x_1,x_3,\omega\,) \tag{3}$$

Here, $u_m(x_1,x_3,\omega)$ denotes the 'mean' translational wavefield in the background homogeneous medium and is given by the well-known solution of the plane SH wave propagation problem for this medium, as

$$u_m(x_1,x_3,\omega)=A_{\mathrm{SH}}(\omega)e^{\left(\frac{\iota\omega}{\beta_m}\sin\theta\right)x_1}\left[e^{\left(\frac{\iota\omega}{\beta_m}\cos\theta\right)x_3}+e^{-\left(\frac{\iota\omega}{\beta_m}\cos\theta\right)x_3}\right] \tag{4}$$

where, $\iota=\sqrt{-1}$, $A_{\mathrm{SH}}(\omega)$ denotes the amplitude of the incident plane SH wave and θ its incidence angle, and $\beta_m=\sqrt{\mu_m/\rho_m}$ denotes the shear wave velocity of the background homogeneous medium. Further, $u_d^{(\mathrm{r})}(x_1,x_3,\omega\,)$ in Eq. (3) denotes the rth perturbation order of 'scattering' wavefield that accounts for the effects of the heterogeneous region. This wavefield results from fictitious body forces $f_r(x_1,x_3,\omega)$ acting in the background homogeneous medium at the locations of the heterogeneities. These body forces depend on the $(r-1)$th perturbation order of heterogeneous wavefield for $r>1$ and on the 'mean' wavefield for $r=1$, as

$$f_r(x_1,x_3,\omega)=\begin{cases}\dfrac{\partial\mu_d}{\partial x_1}\dfrac{\partial u_m}{\partial x_1}+\dfrac{\partial\mu_d}{\partial x_3}\dfrac{\partial u_m}{\partial x_3}+\mu_d\left(\dfrac{\partial^2u_m}{\partial x_1^2}+\dfrac{\partial^2u_m}{\partial x_3^2}\right)+\rho_d\omega^2u_m & r=1\\[2ex] \dfrac{\partial\mu_d}{\partial x_1}\dfrac{\partial u_d^{(\mathrm{r-1})}}{\partial x_1}+\dfrac{\partial\mu_d}{\partial x_3}\dfrac{\partial u_d^{(\mathrm{r-1})}}{\partial x_3}+\mu_d\left(\dfrac{\partial^2u_d^{(\mathrm{r-1})}}{\partial x_1^2}+\dfrac{\partial^2u_d^{(\mathrm{r-1})}}{\partial x_3^2}\right)+\rho_d\omega^2u_d^{(\mathrm{r-1})} & r>1\end{cases} \tag{5}$$

To obtain $u_d^{(\mathrm{r})}(x_1,x_3,\omega\,)$ due to the above body forces, the heterogeneous region is discretized into sufficiently small rectangular elements of lengths Δx_1 and Δx_3 (along the x_1- and x_3-directions respectively), and the contribution from the body force acting at each of these small elements is summed up:

$$u_d^{(\mathrm{r})}(x_1,x_3,\omega\,)=\sum_i f_r\left(x_1^{(i)},x_3^{(i)},\omega\right)g_u\left(x_1,x_3,\omega;x_1^{(i)},x_3^{(i)}\right)\Delta x_1\Delta x_3 \tag{6}$$

Here, $x_1^{(i)}$ and $x_3^{(i)}$ respectively denote the x_1- and x_3-coordinates of the ith heterogeneous element and $g_u\left(x_1,x_3,\omega;x_1^{(i)},x_3^{(i)}\right)$ is the Green's function for translational SH wave at (x_1,x_3) due to the source at $(x_1^{(i)},x_3^{(i)})$. In a closed form, it can be expressed as:

$$g_u\left(x_1, x_3, \omega; x_1^{(i)}, x_3^{(i)}\right) = \left[-\frac{\iota}{4\mu_m}\left\{H_0^{(2)}(\omega r_1/\beta_m) + H_0^{(2)}(\omega r_2/\beta_m)\right\}\right]^* \tag{7}$$

where $r_1 = \sqrt{\left(x_1 - x_1^{(i)}\right)^2 + \left(x_3 - x_3^{(i)}\right)^2}$, $r_2 = \sqrt{\left(x_1 - x_1^{(i)}\right)^2 + \left(x_3 + x_3^{(i)}\right)^2}$, $H_0^{(2)}$ denotes the second Hankel function of order zero, and C^* denotes the complex conjugate of C.

Thus, starting with the mean wavefield $u_m(x_1, x_3, \omega)$ (see Equation (4)), one can use Equations (5) and (6) to recursively determine the scattered wavefield $u_d^{(\mathrm{r})}(x_1, x_3, \omega)$ for any arbitrarily high perturbation order r.

Considering that the surface rotational wavefield $\Omega_3(x_1, \omega)$ is related to the surface translational wavefield $u_2(x_1, \omega)$ as

$$\Omega_3(x_1, \omega) = \frac{1}{2}\frac{\partial}{\partial x_1} u_2(x_1, \omega) \tag{8}$$

the solution of $\Omega_3(x_1, \omega)$ can be obtained by substituting Equation (3) into Equation (8):

$$\Omega_3(x_1, \omega) = \Omega_m(x_1, \omega) + \sum_{r=1}^{\infty} \epsilon^r\, \Omega_d^{(\mathrm{r})}(x_1, \omega) \tag{9}$$

Here, $\Omega_m(x_1, \omega)$ denotes the 'mean' surface rotational wavefield, i.e., the surface rotational wavefield due to propagation of plane SH wave in a homogeneous elastic half-space

$$\Omega_m(x_1, \omega) = \frac{\iota\omega}{\beta_m}\sin\theta\, A_{\mathrm{SH}}(\omega) e^{\left(\frac{\iota\omega}{\beta_m}\sin\theta\right)x_1} \tag{10}$$

and $\Omega_d^{(\mathrm{r})}(x_1, \omega)$ denotes the rth order 'scattered' rotational wavefield given by

$$\Omega_d^{(\mathrm{r})}(x_1, \omega) = \sum_i f_r\left(x_1^{(i)}, x_3^{(i)}, \omega\right) g_\Omega\left(x_1, \omega; x_1^{(i)}, x_3^{(i)}\right) \Delta x_1 \Delta x_3 \tag{11}$$

where

$$g_\Omega\left(x_1, \omega; x_1^{(i)}, x_3^{(i)}\right) = \frac{1}{2}\frac{\partial}{\partial x_1}\left[g_u\left(x_1, x_3 = 0, \omega; x_1^{(i)}, x_3^{(i)}\right)\right]$$

After the calculations have been performed for the whole frequency range, the time-signal can be obtained by applying the inverse Fourier transform.

(12)

Test of Convergence

It may be seen that whereas the series solutions (see Equations (3) and (9)) of the translational and rotational wavefields comprise infinite terms, one needs to evaluate only a finite number of terms as both these series converge for sufficiently high perturbation orders.

This is demonstrated in the case of a simple heterogenous medium model shown in Figure 2. The heterogeneous region in this figure has a square geometry and is uniformly composed of a material with shear modulus μ_m and density ρ. Whereas the shear modulus of the background medium is the same as that of the heterogeneous region, its density ρ_m is different from that of the latter region.

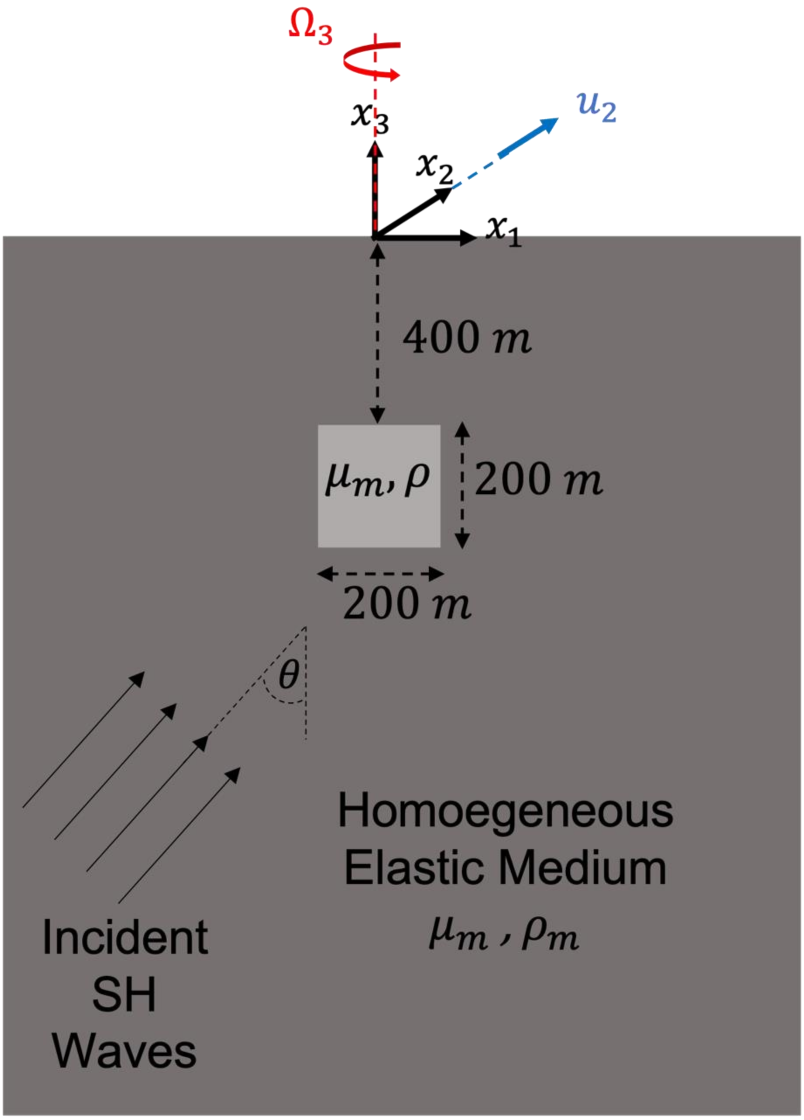

Figure 2: Test heterogeneous medium considered for testing the proposed semi-analytical approach.

The convergence of the proposed solutions is tested by taking the homogeneous material properties as $\rho_m = 2400\ \mathrm{kg/m^3}$ and $\beta_m = 2\ \mathrm{km/s}$ and by considering two shear-wave velocities of the heterogenous region: (1) $\beta = 1.8\ \mathrm{km/s}$ (which corresponds to 10% heterogeneity) and (2) $\beta = 1.6\ \mathrm{km/s}$ (which corresponds to 20% heterogeneity). Further, the angle of incidence θ of the plane wave is taken as 10° and its amplitude $A_{SH}(\omega)$ as unity for all frequencies $f\ (= \omega/2\pi)$.

Figures 3 shows the variations of $\left|\sum_r \epsilon^r u_d^{(r)}(x_1, \omega)\right|$ and $\left|\sum_r \epsilon^r \Omega_d^{(r)}(x_1, \omega)\right|$ with the perturbation order r in the case of 10% heterogeneous model for frequencies $f =$

1, 2, and 4 Hz and at the location $x_1 = 0$. Figure 4 shows these variations in the case of 20% heterogeneous model. In each case, it may be seen that the series converge at an appropriate perturbation order r. However, the value of r at which this happens depends on the frequency as well as the degree of heterogeneity. Figures 5(a) and 5(b) (for the cases of the 10% and 20% heterogenous media, respectively) show the variation of r_{convg} with frequency, where r_{convg} is the perturbation order for which the maximum relative difference between the amplitude and phase spectra of the r_{convg}th and $\left(r_{convg} - 1\right)$th order solutions is less than 0.5%. It may be seen that for a given heterogeneity, the overall trend indicates an increase in r_{convg} with increase in frequency. However, r_{convg} increases (with frequency) at a much faster rate in the case of a strongly heterogenous medium.

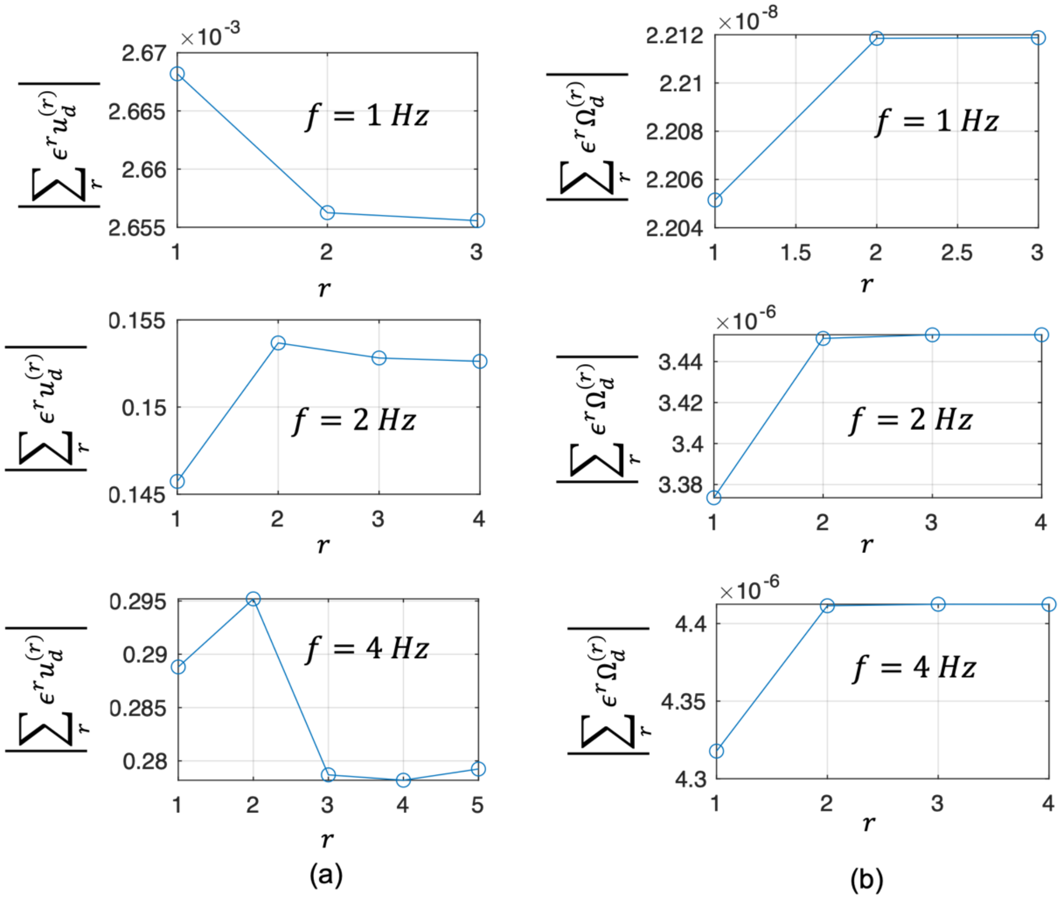

Figure 3: Plots showing convergence (in an absolute sense) of the series solution of (a) translational and (b) rotational scattered wavefields in the case of 10% heterogeneous medium.

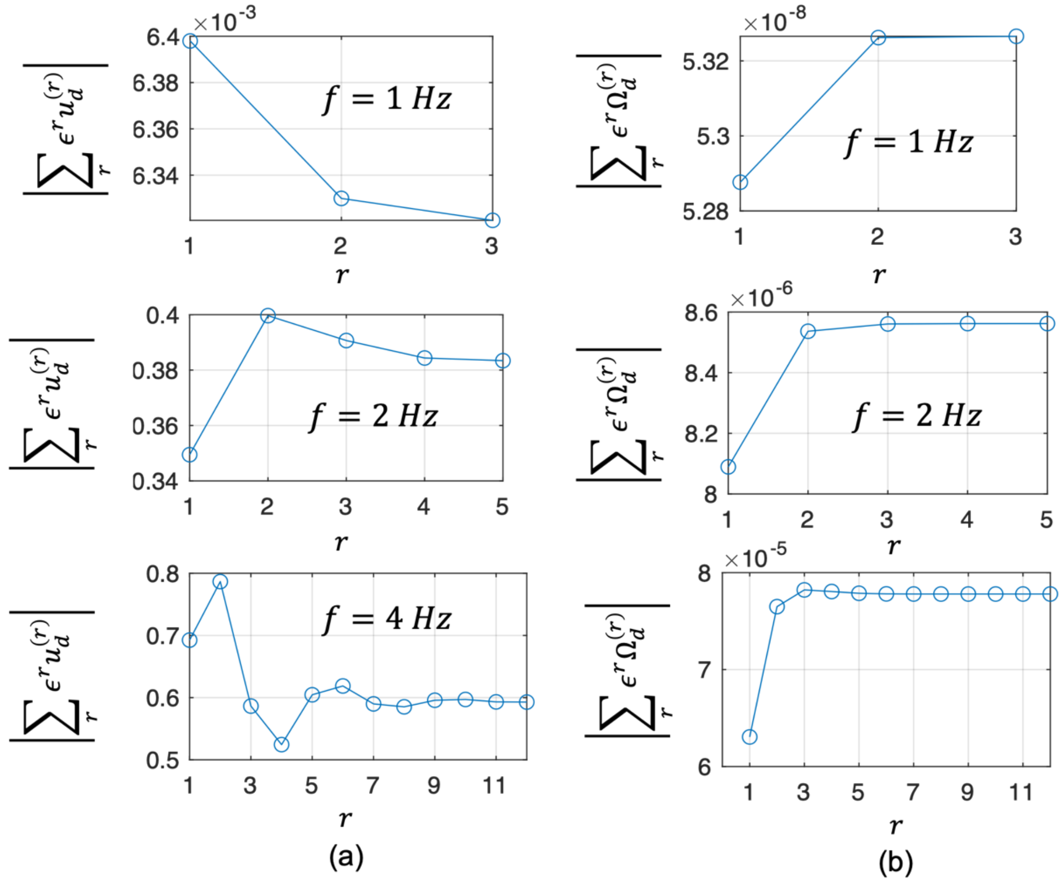

Figure 4: Plots showing convergence (in an absolute sense) of the series solution of (a) translational and (b) rotational scattered wavefields in the case of 20% heterogeneous medium.

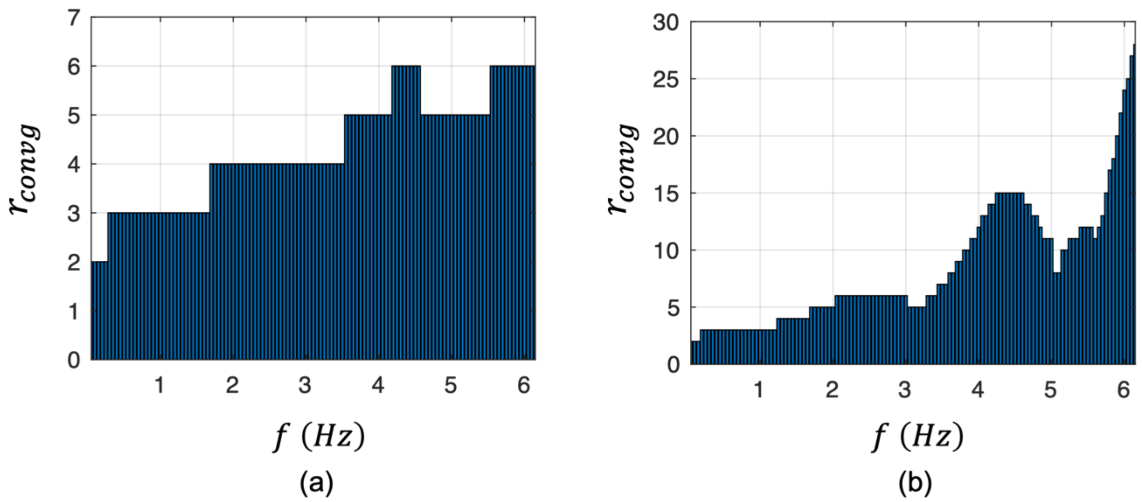

Figure 5: Plots showing variation of the perturbation order for which the series solutions of translational and rotational scattered wavefields converge in the case of (a) 10% and (b) 20% heterogeneous media.

Validation of the Proposed Solution

The proposed semi-analytical solution is next validated in the time domain against the SPECFEM2D solution in the case of the heterogeneous medium considered above (see Figure 2). To simulate the plane waves in SPECFEM2D, a wave propagation model comprising an SH line-source embedded sufficiently deep (or, equivalently, far away from the origin) in the background medium is considered. The vertical and horizonal dimensions of this model are chosen sufficiently large so that there is no contamination of the primary signal with the spurious reflections from the boundaries of the model. The source time function is taken as a ricker wavelet with dominant frequency equal to 2.5 Hz. Further, the time $t = 0$ is made to correspond to the arrival time of the SH wavefront at the origin due to an impulsive line source buried in the corresponding homogeneous medium.

The semi-analytical solution, as outlined above, is first obtained in the frequency domain at discrete frequencies ranging from 0.1 Hz to 7.5 Hz for the case of $\theta = 10°$. The time-domain solution is then obtained using Fourier synthesis. It may be mentioned that the Fourier spectrum $A_{SH}(\omega)$ of the incident SH wave is extracted from the SPECFEM2D translational waveform corresponding to the case of a completely homogeneous medium, for which

$$A_{\mathrm{SH}}(t) = \frac{1}{2}u_2(t) \Rightarrow A_{\mathrm{SH}}(\omega) = \frac{1}{2}u_2(\omega) \tag{12}$$

Figures 6 shows the comparison between translational waveforms (at $x_1 = 0$) obtained using the proposed solution and the SPECFEM2D solver in the case of 10% heterogeneous medium, and Figure 7 show this comparison in the case of 20% heterogeneous medium. Figures 8 and 9 similarly show comparisons of the rotational waveforms corresponding to 10% and 20% heterogeneous media respectively. Here, the SPECFEM2D rotational waveforms are synthesized from the translational motions at $x_1 = -1$ m and $x_1 = +1$ m by using the finite-difference approximation of the Equation (8) expressed in the time domain.

The excellent agreement between the translational as well as rotational waveforms of the proposed method and SPECFEM2D solver, in both types of media, validates the proposed solution.

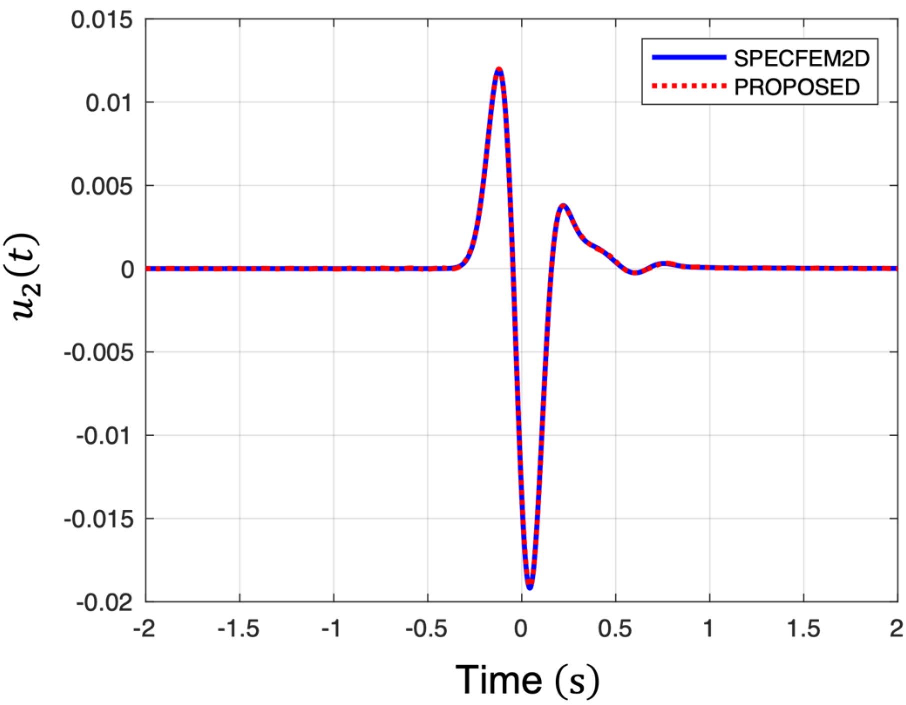

Figure 6: Comparison between translational waveforms obtained under the proposed semi-analytical method and the SPECFEM2D solver in the case of 10% heterogeneous medium.

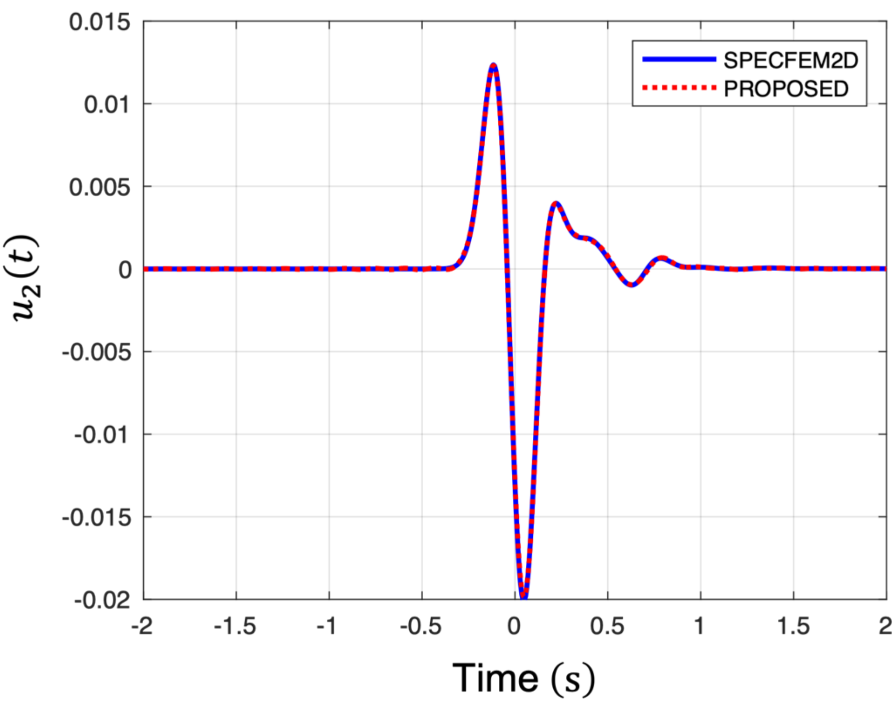

Figure 7: Comparison between translational waveforms obtained under the proposed semi-analytical method and the SPECFEM2D solver in the case of 20% heterogeneous medium.

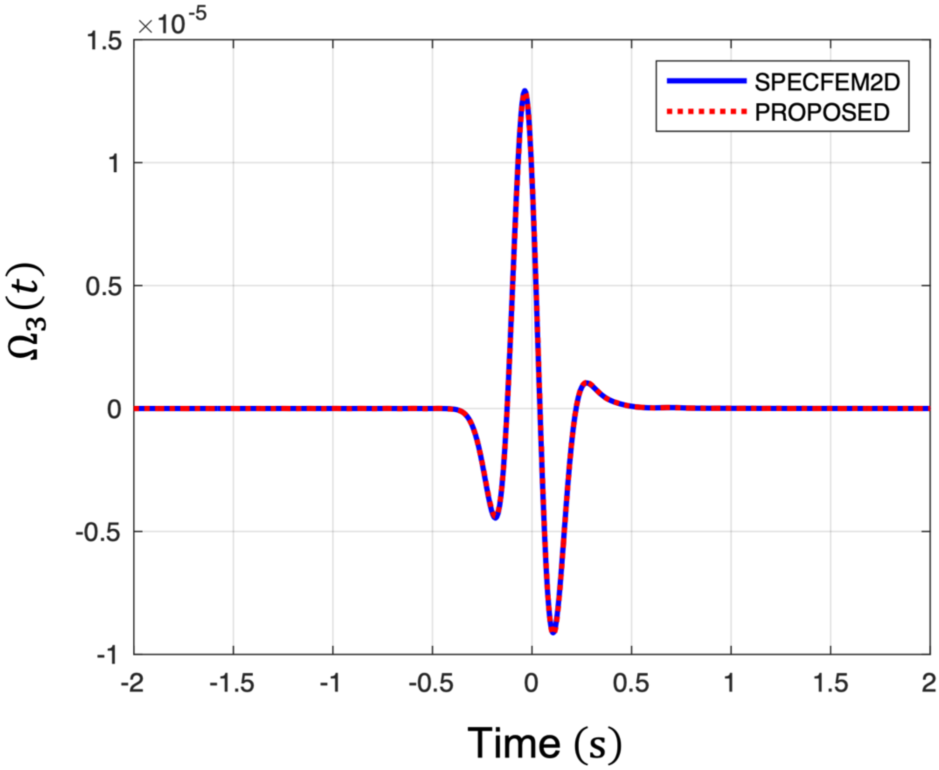

Figure 8: Comparison between rotational waveforms obtained under the proposed semi-analytical method and the SPECFEM2D solver in the case of 10% heterogeneous medium.

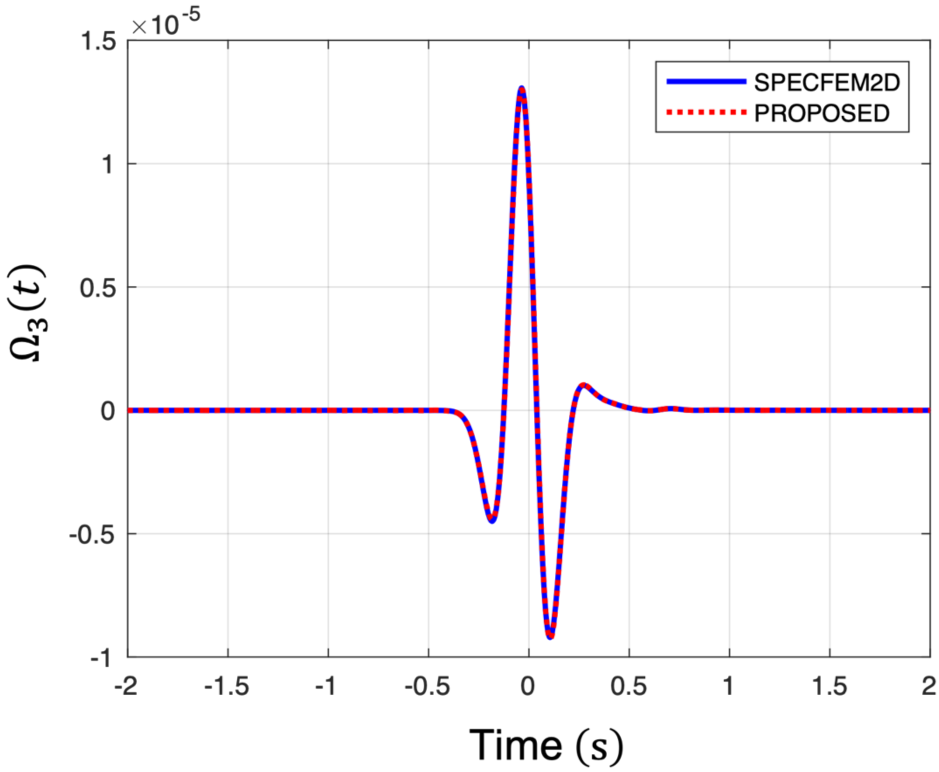

Figure 9: Comparison between rotational waveforms obtained under the proposed semi-analytical method and the SPECFEM2D solver in the case of 20% heterogeneous medium.

Amplifications in Rotational Spectra

The effect of the subsurface heterogeneities on the ground rotations is evaluated in the case of the above-described heterogeneous medium (Figure 2) by computing the percentual difference between the rotational amplitude spectra corresponding to the heterogeneous and homogeneous media, i.e., $100 \times \frac{|\Omega_3(x_1,\omega)|-|\Omega_m(x_1,\omega)|}{|\Omega_m(x_1,\omega)|}$ (see Equations (9) and (10)). Figures 8(a) and 8(b) show the variation of this ratio (at $x_1 = 0$) over various frequencies in the case of the 10% and 20% heterogeneous media respectively. It may be seen that there are amplifications in the rotational spectra, particularly at higher frequencies, and that this amplification is larger in the case of a strongly heterogeneous media. Figures 9(a) and 9(b) show the amplifications in the translational spectra in the case of the two media. On comparing Figures 8 and 9, it may be seen that, for a given heterogeneous medium, the translational spectra are much more amplified than the rotational spectra. However, at high frequencies (>5 Hz), it may be seen that an increase in the heterogeneity impacts rotational amplifications to a greater degree than the translational amplifications (for example, there is a four-fold increase in amplifications of rotational spectra as compared to the two-fold increase in the amplification of the translational spectra). We plan to conduct a similar study for a more complex heterogeneous medium to infer statistically robust conclusions.

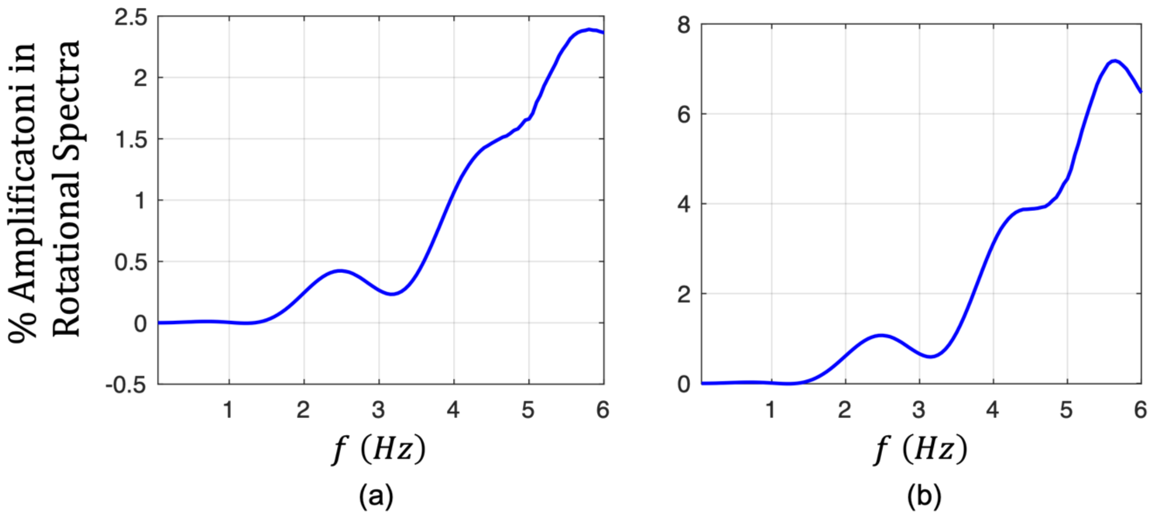

Figure 8: Plots showing % amplifications in rotational spectra in the case of (a) 10% and (b) 20 % heterogeneous media.

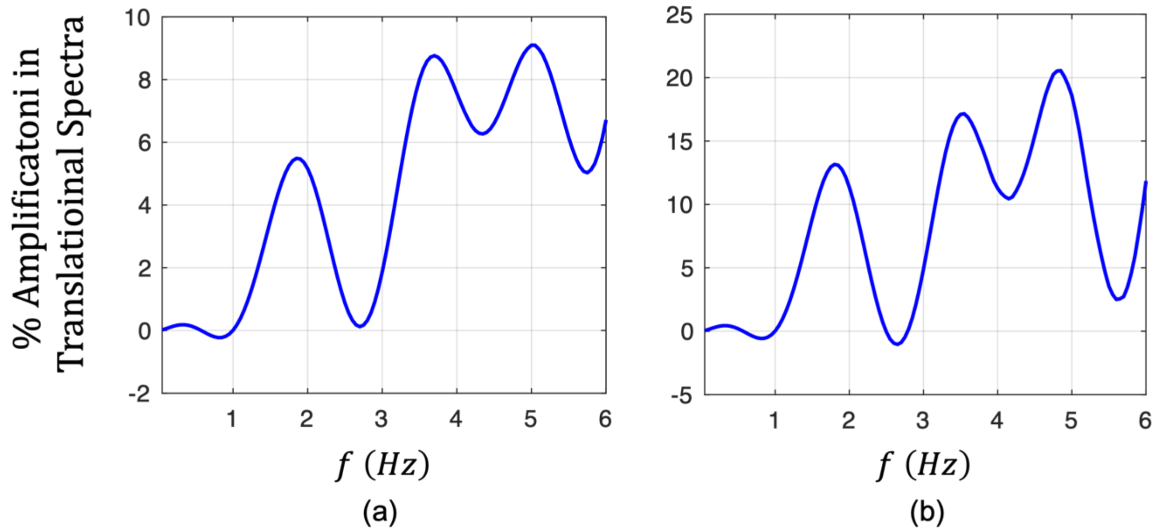

Figure 9: Plots showing amplifications in translational spectra in the case of (a) 10% and (b) 20 % heterogeneous media.

(ii) Implementation (milestones, deliverables, project management)

The project was extended from the originally proposed 6 months to 10.5 months (4.5 months of no-cost extension). Although such a project requires at least a year of active research, the maximum available funds – if spread over a year – were not sufficient to attract a skilled researcher. The 6 months compromise was long enough to develop a concept for the proposed work, but it was not sufficient to fully implement the concept and publish the findings. In addition, the short project duration was negatively impacted by the Covid-19 sick leave of the main researcher - consequently, the project directions and deliverables were consequently adapted to new circumstances (see "deviations" below). Despite all the outlined difficulties, the candidate expressed his interest in conducting further research, attending EGU 2022 and publishing the paper, beyond the timelines of the project. During the no-cost extension period, the candidate simultaneously took on a new job in the role of Assistant Professor at Indian Institute of Technology Roorkee (India), which further impacted the speed of the project progress, but also opened the new avenues for possible future collaborations. In terms of knowledge exchange, regular meetings between PI and the researcher were held throughout the project duration.

(iii) Outputs to date

- Developed semi-analytical solution of the surface translational and rotational Fourier spectra for a 2-D heterogeneous semi-infinite elastic medium excited by plane SH waves (provided above in this Report). This solution has been tested for convergence and has also been validated against SPECFEM2D solution for a simple heterogeneous model.
- Python code for generating an ensemble of 2-D heterogeneous media following

Van Karman spatial variation of heterogeneities (submitted with this Report).

- A working draft of the paper aimed for submission to a journal containing the complete solution of the above-described wave model (this will be completed and sent for publication by February 2022).
- Abstract submission to EGU conference 2022

(iv) Impact

- Relevance/application to GSI programmes, where applicable

- Contributing to scientific knowledge in the field of geohazard
- Opening venues for future collaboration between Irish and international research institutions in the area of geohazard (in particular Indian Institute of Technology Roorkee, where the main researcher on this project got permanent position)

- Other impacts

Every improvement in risk mitigation is significant step forward towards preventing huge human and economic losses worldwide. Although rotational effects of seismic waves, together with rotations caused by soil–structure interaction have been observed for decades and recognised by civil engineering community, they were largely neglected due to the lack of data and inability to model these effects in a medium comprising small-scale heterogeneities over a broad frequency range needed for structural design.
This project aimed to bridge this gap: a relatively simple pseudo-analytical method for calculating rotational ground motion for SH waves in a 2-D heterogeneous medium has been developed. This may have positive impact to the progress in the field, in particular:

- Making the methodology and workflow for calculating SH rotational field in a 2D heterogeneous medium available to wider scientific community through GSI web portal (this report) and future publication
- The concept behind the proposed formulation can be readily applied to the development of similar schemes for P-SV waves, as well as the extension to 3D medium – this could lead to a new class of (potentially efficient) methods to simulate (translational and rotational) ground motions in complex media.
- The method is more suitable for statistical analysis of rotational effects than fully numerical solvers, such as SPECFEM or similar, this providing a good basis for seismic hazard assessment in urban areas.
- This is the first effort to address the question of rotational amplification due to small-scale heterogeneities, simultaneously with the advent (but still very sparse use) of the portable instruments for measuring ground rotations.

Publications:
- in preparation
- EGU abstract to be submitted before Christmas